Homework 2

Due 1/21/2010

1. [5 points] Consider a D-dimensional Bravais lattice, with an arbitrary natural number D, generated by D linearly independent vectors $\vec{a}_1, \vec{a}_2, \dots \vec{a}_D$, as $\vec{R} = n_1\vec{a}_1 + n_2\vec{a}_2 + \cdots + n_D\vec{a}_D$ with $n_1, \dots, n_D \in \mathbb{Z}$ (the set of integers). Then, the definition of its reciprocal lattice $\{\vec{G}\}$ can be generalized (following what we did in class) as

$$\vec{a}_i^* \cdot \vec{a}_j = 2\pi\delta_{ij} \quad \cdots \quad (1) \qquad i, j = 1, \dots, D$$

$$\vec{G} = m_1\vec{a}_1^* + m_2\vec{a}_2^* + \cdots + m_D\vec{a}_D^*, \qquad m_i \in \mathbb{Z}$$

   Now consider two $D \times D$ matrices

$$J_{orig} = [\vec{a}_1 \quad \cdots \quad \vec{a}_D], \quad J_{reci} = [\vec{a}_1^* \quad \cdots \quad \vec{a}_D^*]$$

   where each vector in the square brackets should be understood as a D dimensional *column* vector. By multivariable calculus, the determinant of J_{orig} is the signed volume, V, of the unit cell of the original Bravais lattice and the determinant of J_{reci} is the signed volume, V^*, of the unit cell of the reciprocal Bravais lattice. Using the above definition (1), show that

$$\boxed{VV^* = (2\pi)^D}$$

   This is the general form of what was stated in class for $D = 3$, without proof.
   Hint: Well known properties of the determinant: $\det(A) = \det(A^t)$, $\det(AB) = \det(A)\det(B)$.
   Note 1: The sign for V and V is determined by the relative orientations of the vectors* $\{\vec{a}_i\}$.
   Note 2: Why would one consider D greater than 3 at all? It is not just for mathematical curiosity, power, and satisfaction. The generalization to higher dimensions than 3 can be of practical use. For instance, some materials, a certain kind of "quasi-crystals," are known to have a non-crystalline structure that is the projection of a six dimensional crystalline structure onto a three dimensional sub-space.

2. [5 points] Kittel 2.2

3. [5 points] In order to satisfy the Bragg condition, the wavelength of a quantum particle that diffracts off of the crystal needs to be on the order of, or smaller than, the spacing of lattice planes. Since the typical lattice spacing of a crystal is a few Å, the wavelength of about 1 Å can be taken as a typical required value. Find corresponding energies in unit of eV (electron volt) when the probing particle is (a) the photon, (b) the electron, and (c) the neutron. (d) Which particle can be described as “thermal” in the sense that its energy is on the order of the room temperature 300 K (Note that, by the equipartition theorem, the thermal energy $\sim k_B T$). You may find useful the "Physical constants" sheet that was distributed in class.

4. [10 points] Kittel 2.~~3~~ 4

5. [10 points; extra credit] Kittel 2.1